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Short-Range Gravitational Field and the Red Shift of Quasars

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A fourth-order gravitational field equation is used for the case of a static field. Useful expressions for the scalar curvature and the potential are obtained by assuming the space to be nearly flat. The expression for the potential shows the existence of a short-range gravitational field and the possibility of explaining the large red shift of quasars.

1. INTRODUCTION

Einstein's theory of general relativity (GR) is regarded as one of the great achievements in physics, as it gives a correct description for a number of gravitational phenomena. In particular, it was tested to be in agreement with experiment in describing the behavior of gravity within the solar system. Despite these advantages, GR suffers from certain noticeable setbacks, especially in the area of strong gravity. For example, in recent decades a new species of astronomical exotica have been discovered, including quasars and pulsars, where a strong gravitational field is assumed to be dominant (Bowler, 1976). The nature of these objects is difficult to understand in terms of GR. For instance, it is difficult to interpret the large red shift of quasars $z \sim 3$ as being due to the gravitational red shift, since the Newtonian potential restricts z (Weinberg, 1972) to the upper bound

$$z < 2$$

and its interpretation, in terms of the cosmological expansion, is rather ambiguous (Weinberg, 1972).

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Various attempts at constructing an alternative model capable of describing strong-gravity phenomena have been made (Ranch, 1982). The most promising one is the generalized field equation (GFE), which is based on the same geometrical description as Einstein's model, but with higher differential order (Laczos, 1969; Ali, 1992). The GFE, besides sharing all the successes of GR in the weak-gravity regions (Xu *et al.*, 1992), is advantageous by generally being nonsingular in regions where gravity is strong (Ali, 1992).

In this paper, by restricting ourselves to the static isotropic metric and a Lagrangian which consists of a quadratic term together with the linear one, we develop a gravitational model which allows short-range solutions. These solutions are utilized to explain the large red shift of quasars.

2. THE GENERALIZED FIELD EQUATION IN STATIC SPHERICALLY SYMMETRIC FIELD

The GFE (Lanczos, 1932; Ali, 1987), which is a fourth-order equation, takes the form

$$\mathscr{L}'''(R_{;\mu}R_{;\nu} - g_{\mu\nu}g^{\rho\sigma}R_{;\rho}R_{;\sigma}) + \mathscr{L}''(R_{;\mu;\nu} - g_{\mu\nu}\Box^2 R) + \mathscr{L}'R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}\mathscr{L} = 0 \tag{1}$$

Contracting this equation yields

$$\Box^2 R = g^{\rho\sigma}R_{;\rho;\sigma} = \frac{\mathscr{L}'R - 2\mathscr{L}}{3\mathscr{L}''} - \frac{\mathscr{L}'''}{\mathscr{L}''}g^{\rho\sigma}R_{;\rho}R_{;\sigma} \tag{2}$$

Assuming the field of a star to be static, the metric is then given by

$$g_{rr} = A(r), \qquad g_{\theta\theta} = r^2, \qquad g_{\phi\phi} = r^2\sin^2\theta, \qquad g_{tt} = -B(r)$$

where the proper time interval is given by

$$ds^2 = B(r)\,dt^2 - A(r)\,dr^2 - r^2\,d\theta^2 - r^2\sin^2\theta\,d\phi^2 \tag{3}$$

Substituting equation (3) in (1), we get the following relations (Ali, 1992):

$$B(r) = \frac{c^2 A}{r^4\dot{R}^2\mathscr{L}''^2}\exp\left[\frac{2}{3}\int A(\mathscr{L}'R - 2\mathscr{L})(\dot{R}\mathscr{L}'')^{-1}\,dr\right] \tag{4}$$

$$A(r) = \frac{12(d/dr)\ln[\mathscr{L}'(r\dot{R}\mathscr{L}'')^{-1}]}{\{3rR + 12/r - (R\mathscr{L}' - 2\mathscr{L})[4/\dot{R}\mathscr{L}'' - r/\mathscr{L}']\}} \tag{5}$$

and

$$\dot{R} = \frac{c}{r^2}\sqrt{\frac{A}{B}}\frac{1}{\mathscr{L}''}\exp\left[\int\frac{A(\mathscr{L}'R - 2\mathscr{L})}{3\dot{R}\mathscr{L}''}\,dr\right] \tag{6}$$

where c is an arbitrary constant. These equations express the metric components in terms of r, $R(r)$, $\mathscr{L}$, and their derivatives. Since the GFE is very complex and highly nonlinear, it is difficult to obtain exact solutions. For the sake of simplicity, let us take the nonlinear Lagrangian

$$\mathscr{L} = -\alpha R^2 + \beta R + \gamma \tag{7}$$

In this case the contracted equation (2) reduces to

$$\Box^2 R = \frac{1}{A}\left[\ddot{R} - \dot{R}\left(\frac{\dot{A}}{2A} - \frac{\dot{B}}{2B} - \frac{2}{r}\right)\right] = \frac{\beta}{6\alpha}R + \frac{\gamma}{3\alpha}$$

$$\left(\Box^2 - \frac{\beta}{6\alpha}\right)R = \frac{\gamma}{3\alpha} \tag{8}$$

where γ is assumed to represent the source term. A further simplification can be achieved by assuming the space to be nearly flat, i.e.,

$$A \to 1, \qquad B \to 1 \tag{9}$$

Equation (8) thus becomes

$$\ddot{R} + \frac{2}{r}\dot{R} = \frac{\beta}{6\alpha}R + \frac{\gamma}{3\alpha} \tag{10}$$

If we are outside the source, $\gamma = 0$, and by setting $\beta = 0$, then equation (10) reduces to

$$\frac{d\dot{R}}{dr} + \frac{2}{r}\dot{R} = 0 \tag{11}$$

therefore

$$\frac{d\dot{R}}{\dot{R}} = -\frac{2}{r}dr \tag{12}$$

Integrating both sides yields

$$\dot{R} = \frac{c}{r^2}$$

and hence

$$R = -\frac{c}{r} + c_1$$

When we are far away from the course the space is flat, i.e., $R \to 0$ as $r \to \infty$, and as a result $c_1 = 0$. The scalar curvature is thus given by

$$R = -\frac{c}{r} \tag{13}$$

Using equations (12) and (7) and the expression for A in a weak field, i.e.,

$$A = \left(1 - \frac{2MG}{r}\right)^{-1} \tag{14}$$

in equation (4) yields

$$B = \frac{A}{4\alpha^2} \exp\left(\frac{\beta}{6\alpha MG} \int r^2 dr\right) = \frac{A}{4\alpha^2} \exp\left(\frac{\beta}{18\alpha MG} r^3\right) = 1 + 2\phi \tag{15}$$

This expression indicates the existence of a short-range gravitational field. If we consider a field near the surface of a small-radius supermassive star, then $r \to 0$, and

$$B = \frac{A}{4\alpha^2} \tag{16}$$

The red shift then becomes

$$z = B^{-1/2} - 1 = 2\alpha\left(1 - \frac{2MG}{r}\right)^{1/2} - 1 \tag{17}$$

since (Weinberg, 1972) $MG/R < 4/9$ and if $\alpha = 6$. Therefore, for the maximum value of this ratio, z is given by

$$z = 2\alpha(1/3) - 1 = 3 \tag{18}$$

An alternative approach can also lead to the same result by seeking a general solution for equation (10). For instance, let

$$R = \frac{c_1}{r} \exp c_2 r + R_0 \tag{19}$$

A direct substitution in equation (10) yields

$$\frac{c_1 c_2^2}{r} \exp C_2 r = R_0 + \frac{\beta c_1}{6\alpha r} \exp C_2 r + \frac{\gamma}{3\alpha}$$

$$c_2 = \pm\sqrt{\frac{\beta}{6\alpha}}, \qquad R_0 = -\frac{\gamma}{3\alpha}, \qquad R = \frac{c_1}{r} \exp - \sqrt{\frac{\beta}{6\alpha}}\, r - \frac{\gamma}{3\alpha} \tag{20}$$

To relate the potential to R, we suppose the metric to be close to the Minkowskian metric, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{21}$$

where we raise and lower indices using $\eta^{\mu\nu}$ as long as we restrict ourselves to first-order in h; therefore (Carmeli, 1982)

$$\begin{aligned} R &= g^{\mu\kappa}g^{\lambda\nu}R_{\lambda\mu\nu\kappa} = g^{00}g^{ii}R_{i0i0} \\ g^{ii} \to \eta^{ii}, &\qquad g^{00} = \eta^{00}\eta^{00}g_{00} = -(1 + 2\phi) \\ R_{i0i0} &= \tfrac{1}{2}\nabla^2 g_{00} = -4\pi G\rho \\ R &= 8\pi G\rho\phi + 4\pi G\rho \end{aligned} \tag{22}$$

Comparing equations (19) and (21) yields

$$\frac{\gamma}{3\alpha} = -4\pi G\rho, \qquad \phi = \frac{c_1}{8\pi G\rho r} \exp - \sqrt{\frac{\beta}{6\alpha}}\, r \tag{23}$$

This indicates again the existence of a short-range force or a possible link with strong nuclear force. If we set

$$C_1 = 8\pi G\rho$$

then the red shift becomes

$$z = (B)^{-1/2} - 1 = \left[1 + \frac{2}{r}\exp\left(-\sqrt{\frac{\beta}{6\alpha}}\, r\right)\right]^{-1/2} - 1$$

$$\approx \left(\frac{r}{2}\right)^{1/2} \exp\left(\frac{1}{2}\sqrt{\frac{\beta}{6\alpha}}\, r\right) - 1$$

when we are just outside the star $\rho = 0$ and one of the possible ways to do this is to set $1/\alpha \to 0$, and for $r = 32$ (Weinberg, 1972)

$$z \approx 3$$

Thus the origin of the large red shift of quasars can be explained.

3. CONCLUSION

Expression (22) shows the possibility of having either a short-range gravitational field or a link between gravity and the short-range nuclear force, and this raises a hope for unifying gravity with the rest of physics. The success of this model in predicting a large red shift for supermassive stars indicates that the observed large red shift can be interpreted as being due to the gravitational red shift. This may open a new horizon to explain the behavior of these exotic objects within the framework of the GFE.

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REFERENCES

Ali El-tahir (1987). A generalized variational principle of gravitation, Preprint IC/87/243, ICTP, Trieste.

Ali El-tahir (1992). *International Journal of Modern Physics A*, **7**, 3133.

Bowler, M. G. (1976). *Gravitation and Relativity*, Pergamon Press, Oxford, Chapter 10.

Carmeli, M. (1982). *Classical Fields, General Relativity and Gauge Theory*, Wiley, New York, Chapter 5.

Kilimister, C. W. (1962). *Les Theories Relativistes de la Gravitation*, Centre National de Recherche Scientifique, Paris.

Lanczos, C. (1932). *Physical Review*, **31**, 716.

Lanczos, C. (1969). *Journal of Mathematical Physics*, **10**, 1057.

Neville, D. E. (1982). *Physical Review D*, **26**, 2638.

Ranch, R. T. (1982). *Physical Review D*, **26**, 9319.

Stephenson, G. (1958). *Nuovo Cimento*, **9**, 263.

Weinberg, S. (1972). *Gravitation and Cosmology*, Wiley, New York, Chapter 11.

Xu Chongming Ellis, G. F. R., Wu Xuejun, and Ali El-tahir (1992). *South African Journal of Physics*, **15**, 5.